Abstract—This paper presents a new method to automatically transport objects with mobile robots via non-prehensile actions. Our proposed approach utilizes a pair of nonholonomic robots connected by a deformable tube to efficiently manipulate objects of irregular shapes toward target locations. To autonomously perform this task, we develop a local integrated planning and control strategy that solves the problem in two steps (viz. enveloping and transport) based on the model predictive control (MPC) framework. The deformable underactuated system is simplified by a linear kinematic model. The enveloping problem is formulated as the minimization of multiple criteria that represent the enclosing error of the object by the variable morphology system. The transport problem is tackled by formulating the non-prehensile dragging action as an inequality constraint specified by the body frame of the deformable system. Reactive obstacle avoidance is ensured by a maximum margin-based term that utilizes the system’s geometry and the feedback proximity to the environment. To validate the performance of the proposed methodology, we report a detailed experimental study with vision-guided robotic prototypes conducting multiple autonomous object transport tasks.

Index Terms—Nonprehensile manipulation; Motion control; Object transport; Deformable agents; Nonholonomic systems.

I. INTRODUCTION

Manipulation of rigid objects has a rich history in robotics, where many important results have been achieved in the last few years. Traditional approaches—relying on fixed-base robot arms with customized end-effectors—can only be used with objects that satisfy special conditions. For example, geometries suitable for grasping [1] or flat surfaces for suction lifting [2]. Mobile robots can perform manipulation tasks without the need to use end-effectors [3], e.g., they can transport objects by dragging them through deformable ropes or cables. This type of (non-prehensile) strategy is useful in alleviating some of the drawbacks of traditional approaches, as it enables to manipulate objects with un-graspable geometries, increases the operation range of the transportation task, and combines the power of multiple robots to move larger loads, among others. Consequently, these types of strategies are potentially useful in many important applications, such as clearing debris from water [4] (Fig. 1. (a)), transporting materials in construction sites [5], or even in domestic cleaning tasks [6]. Despite its evident practical value, the development of non-prehensile controls that exploit deformable materials to indirectly manipulate objects is still an understudied problem in the robotics community.

Many researchers have studied the automatic transportation of objects by mobile agents considering several types of configurations (see [7] for a comprehensive literature review). For agents equipped with arms or grippers, the task can be collaboratively performed by the robot fleet through formation control strategies [8]. This approach was used in [9], [10], where motion planning algorithms were developed to manipulate both rigid and deformable objects with multi-robot systems. These strategies have also been formulated using constrained optimization subject to kinematic/dynamic constraints. For simple agents that lack active grippers, a common method to transport the load is to physically enclose it with the robots’ structure while maintaining a fixed formation throughout the path to ensure a stable mobile grasp [11]. In contrast to gripper-based approaches, the use of dragging/pushing enables the transport of objects with fewer and mechanically simpler robots. A key challenge with these strategies is the presence of uncertain frictional forces [12], which complicate the object’s positioning. Several works have tried to address this issue. For example, [13] proposes a model predictive control (MPC) for nonholonomic robots taking into consideration the constraints associated with the pushing direction, [14] describes an algorithm to compute the optimal contact points to ensure the object’s maneuverability along the path, [15] presents a complete behavioral, reactive, and centralized framework for a multi-agent system to push a set of objects, [16] addresses the transport of non-rigid loads by using differentiable soft-body physics engines, to name a few instances. These previous works have established a solid foundation for the problem, however, more research is needed to develop non-prehensile models/controls for mobile agents composed of deformable elements. The aim of this paper is...
to provide a solution to this open problem.

To perform the task, our proposed system is composed of two nonholonomic mobile bases linked by a flexible tube that provides a large contact area (or multiple interaction points) with the object. This feature enhances the system’s manipulation capabilities, yet its non-rigid nature complicates the development of planning and control algorithms. Note that most existing methods are designed for rigid systems with a fixed morphology [17]. To deal with this non-rigid nature, this paper proposes a new integrated planning and control strategy based on the MPC framework for deformable mobile agent systems, which provides efficient local motion control strategy based on the MPC framework for deformable nature, this paper proposes a new integrated planning and control algorithms. Note that, to get the middle point of the tube directly, the number \( m \) must be odd. We model the curved geometry of deformable tube as an unknown nonlinear function \( q = f(\chi) \), where \( q = [q_1^T, q_2^T, \ldots, q_m^T]^T \in \mathbb{R}^{2m} \) denotes the position of the feature points, and \( \chi = [x_1, y_1, x_2, y_2]^T \in \mathbb{R}^4 \) represents the feedback position of the mobile bases. With this function, we obtain the following shape-motion model of the system:

\[
q = \frac{\partial f}{\partial \chi} \chi = J(\chi)\dot{\chi} \tag{2}
\]

for \( J(\chi) \in \mathbb{R}^{2m \times 4} \) as the Jacobian matrix, whose analytical computation requires knowledge of the physical parameters of the tube, which are hard to obtain in practice. This structure can be numerically estimated from feedback data [18].

Different from the existing research, the proposed system has little requirement for the shape of the objects due to the application of the deformable tube. The contact point or surface of contact between the tube and the target object is not predetermined strictly, which holds greater significance when dealing with objects of irregular shapes. Thus, as shown in the left subfigure of Fig. 2, the target object can be represented by its minimum enclosing circle in 2-D. The position of the target object is defined as the center point \( p_i \) of the minimum enclosing circle with the radius \( r_i \).

To explain the enveloping task, the virtual geometric center point \( p^v \) of the proposed system (shown in the left subfigure of Fig. 2)) is given, which is defined as

\[
p^v = \frac{1}{m+2} \left( \sum_{i=1}^{2} x_i + \sum_{j=1}^{m} q_j \right) \tag{3}
\]

where \( x_i = [x_i, y_i]^T \) is the position of the mobile robot. We note that the proposed system can only handle the target object within a certain size that can be represented by:

\[ r_t < \min (\|q_j - p^v\|, \|\chi_i - p^v\| - r_e) \tag{4} \]
where $r_e$ is the radius of the end mobile robot. Besides, considering that this system can only envelop the target from its “open” side that has no tube connected, the relative orientation of the system to the target object is considered and regarded as another target of the optimization. As shown in the right subfigure of Fig. 2, the direction of the system (the green arrow) is defined as the vector from the middle feature points $q_M$ of the tube to the virtual middle point $\chi_M$ of the two end robots, while the yellow arrow shows the relative direction between the object and the flexible mobile agent system. Then, the relative orientation $\theta$ is defined as the angle between these two vectors, which can not be obtained by the camera directly and is calculated by

$$\theta = \arccos \left( \frac{r_1 \cdot r_2}{||r_1|| \cdot ||r_2||} \right) \quad (5)$$

where $r_1 = p_t - q_M$, $r_2 = \chi_M - q_M$, with $\chi_M = (x_1 + x_2) / 2$ and $q_M = q_{m+1}/2$. Then, the enveloping task can be defined as follows.

**Problem 1.** Develop an MPC-based control method for the flexible mobile agent system (1)–(2), to automatically envelope an irregular-shaped object (i.e., flexible mobile agent system (1)–(2), to automatically deliver the transported object into the receptacle for the whole transport system described by (1) and (6), to reactivity avoid any collisions with the object, obstacles, and the bounded environment.

### C. Transport Problem Statement

As for the transport process, the challenge is to control the motion of the transported object pulled by the tube to the target receptacle. The transported object and the flexible mobile agent system are regarded as one new transport system. We assume that inertial forces are negligible or quickly absorbed by the frictional effects, i.e., quasi-static assumption. Due to the slip between the tube and the transported object and the complex physical properties of the tube, the whole transport system has a highly unstable and nonlinear dynamic. For this new system, the position of the object $p_t$ is regarded as part of the state instead of the shape of the tube. Similarly, we define the kinematics model of the transported object as

$$\dot{p}_t = J_t(\chi) \dot{\chi} \quad (6)$$

where $J_t(\chi) \in \mathbb{R}^{2 \times 4}$ represents how end mobile robots change the motion of the transported object during transport. The matrix $J_t(\chi)$ must be updated in real time because the object is not fixed to the tube and may slide along it. We still use the update algorithm in [18] to update the matrix $J_t(\chi)$ using the visual feedback information.

The target receptacle is given as a rectangle zone $R_z$, of which the geometry center is represented by $T_z \in \mathbb{R}^2$. Then, the transport problem can be given as:

**Problem 2.** Develop an MPC-based local control method for the whole transport system described by (1) and (6), to automatically deliver the transported object into the receptacle (i.e., $p_t \in R_z$), while avoiding collisions.

### III. METHODOLOGY

In our framework, the optimization problems of two processes (the envelope and the transport) are formulated, and the real-time update methods of the MPC algorithm are given to ensure the performance of the optimization and obstacle avoidance.

#### A. Enveloping Problem Formulation

To control the flexible deformable mobile agent system to envelop the target object without any collision with the obstacle and the target object in the bounded environment, we present a novel MPC formulation to calculate the control input $u_i$ of the two end robots, as shown in Fig. 3. The finite forecast horizon is set as $n$ steps with the sampling time $T_s$. This MPC is constrained by (i) dynamics (1)–(2), (ii) bounds on control input, (iii) position bounds, (iv) distance range between the two end robots, (v) collision avoidance with the environment, and (vi) obstacle avoidance with the environment. And (v) and (vi) are designed as soft constraints in the objective function. The optimal tasks contain the control input, the distance error, and the relative orientation. Then, for the predicted state estimate at each discrete-time instant $k$, this multi-task constrained optimization problem is formulated as

$$\min_{u} F = \sum_{k=1}^{n} (\alpha_u F_u[k] + \alpha_d F_d[k] + \alpha_\theta F_\theta[k])$$

s.t. \quad $u_{\min} \leq u[k] \leq u_{\max}$ \quad (7a)

$$\chi_{\min} \leq \chi[k] \leq \chi_{\max}$$

$$q_{\min} \leq q[k] \leq q_{\max}$$

$$d_{\min} \leq \|X_1[k] - X_2[k]\| \leq d_{\max}$$ \quad (7b)

where $\alpha_u, \alpha_d, \alpha_\theta, \alpha_o$ are the weights of each term, $u_{\min} \in \mathbb{R}^4$, $u_{\max} \in \mathbb{R}^4$, $\chi_{\min} \in \mathbb{R}^4$, $\chi_{\max} \in \mathbb{R}^4$, $q_{\min} \in \mathbb{R}^{2m}$, $q_{\max} \in \mathbb{R}^{2m}$ are all constant vectors, that give the range of variables $u$, $\chi$, $q$, respectively. While $d_{\min}$ and $d_{\max}$ are the positive constants, which gives the distance range between the two end robots. The objective function (7a) is comprised of several terms, each of which is explained as follows:

- **a) Control input term.** To save the energy of the system, the control input term $F_u$ is formulated as

$$F_u[k] = u[k]^T Q_u u[k]$$ \quad (8)

where $u[k] = [v_1[k], \omega_1[k], v_2[k], \omega_2[k]]^T \in \mathbb{R}^4$ is the predicted control input (velocity and angle velocity) of two end mobile agents at $k$-th step, $Q_u \in \mathbb{R}^{4 \times 4}$ is the positive definite matrix that can adjust the proportion of velocity $v_i$ and angular velocity $\omega_i$ of two end mobile robots.
where \( K_d \in \mathbb{R}^{2 \times 2} \) is the positive definite matrix that can modify the ratio among various directions, \( \mathbf{p}^v[k] \) is the predicted virtual geometric central point at \( k \)-th step.

**c) Steer term.** To realize the control of the relative orientation, the steer term \( F_{\theta}[k] \) is formulated as \(-c\cos\theta[k]\). The calculation of the cosine factor is more straightforward, and it also represents the error between the orientation of the system and the direction needed to enclose the target object. The steer term can drive \( \theta \) to zero, indicating that the target object is completely situated on the “open” side of the system.

One major challenge in optimizing a multi-task model is the conflicting gradients, which impact the performance of specific tasks [19]. To ensure the optimization of both targets (the distance error and the relative orientation), it is crucial to design the weights of targets (the distance error and the relative orientation), it is of specific tasks [19]. To ensure the optimization of both targets (the distance error and the relative orientation), it is crucial to design the weights of these two terms along this side of the system. Thus, before each iteration of the optimization, the weight \( \alpha_{\theta} \) undergoes an update by the following rule to ensure that both targets have an equal influence on the direction of the objective function’s optimization.

\[
\alpha_{\theta} = \frac{\alpha_d}{2} \sum_{i=1}^{2} \left( \frac{\|F_d\|}{\|\partial F_d/\partial X_i\|} \right) \quad (10)
\]

The gradient of \( F_{\theta} \) with respect to \( X_i \) is calculated by:

\[
\frac{\partial F_{\theta}}{\partial X_i} = z_2^T \frac{\partial z_1}{\partial X_i} + z_1^T \frac{\partial z_2}{\partial X_i} = z_2^T \frac{\partial z_1}{\partial X_i} + z_1^T \frac{\partial z_2}{\partial X_i} \quad (11)
\]

\[
= z_1^T \left( I_2 - z_2z_2^T \right) \frac{\partial r_2 - r_2}{\|r_2\|} \frac{\partial r_1}{\|r_1\|} + z_2^T \left( I_2 - z_1z_1^T \right) \frac{\partial q_M}{\|r_2\|} \frac{\partial q}{\|r_1\|}
\]

where \( z_1 = r_1/\|r_1\|, z_2 = r_2/\|r_2\|, \) and matrix \( J^M \in \mathbb{R}^{2 \times 2} \) is the block of of \( J(\chi) \) at the \((2i-1)^{th},(2i)^{th}\) and middle two rows. The main difference between \( \frac{\partial F_d}{\partial X_i} \) and \( \frac{\partial F_{\theta}}{\partial X_i} \) are the matrix \( J^M_i \), which depends on the physical properties of our system. Considering the symmetric configuration of our system, it is assumed that the \( \frac{\partial F_d}{\partial X_i} \) and \( \frac{\partial F_{\theta}}{\partial X_i} \) are nearly equal. This assumption also works for the gradient of \( F_d \) with respect to \( \chi_i \), which can be obtained by:

\[
\frac{\partial F_d}{\partial \chi_i} = \frac{\partial F_d}{\partial p^v} \frac{\partial p^v}{\partial \chi_i} = (p^v-p_i)^T \left( K_d^T + K_d \right) \frac{\partial p^v}{\partial \chi_i} \quad (12)
\]

where \( J_i \in \mathbb{R}^{2 \times 2} \) is the block of \( J(\chi) \) at the \((2i-1)^{th},(2i)^{th}\) columns and \((2j-1)^{th},(2j)^{th}\) rows.

d) Target collision penalty term. It is imperative to avoid any contact between the system and the object while executing the enveloping task. Similar to the repulsive potential field utilized in the artificial potential field approach [20], the collision penalty term is designed as:

\[
F_{\theta} = \sum_{i=1}^{2} F_{\theta}^d(\chi_i[k]) + \sum_{j=1}^{m} F_{\theta}^d(q_j[k]) \quad (13)
\]

in which \( k_i > 0 \) is the repulsive gain, and \( r_0 \) is the allowed minimal distance between the mobile system and the target object. It is a soft constraint, which is equivalent to the inequality \( \|\mathbf{q} - \mathbf{p}_i\| \geq r_0 \). For feature points \( \mathbf{q}_j \), \( r_0 \) can be set as the radius \( r_1 \) of the minimum enclosing circle. For the end mobile robot \( \mathbf{x}_i \), considering the radius of the robot, \( r_0 = r_1 + r_c \). Besides, the weight \( \alpha_i \) is designed to be large enough to ensure that this soft constraint will not be violated.

**e) Obstacle avoidance term.** The shape of the proposed system must be taken into consideration as our system cannot be perceived as a singular entity. The system is estimated to be a convex polygon \( \text{conv} = \{ \chi_1, \chi_2, \ldots, \chi_n \} \). And, the perceived obstacle information is donated by a point set \( O = \{ \mathbf{o}_i \mid i = 1, 2, \ldots \} \), in which \( \mathbf{o}_i \) is the vertex or the point on the boundary of the obstacle sensed by robots. Hence, the max-margin method is introduced as a solution to avoid the obstacle, drawing inspiration from the supporting vector machine [21]. The following constrained quadratic programming problem obtains the hyperplanes with the max-margin between \( \text{conv} \) and \( O \):

\[
\min \frac{1}{2} \|w\|^2, \quad \text{s.t.} \quad z_i (w^T \mathbf{p}_i + b) \geq 1, \quad \forall \mathbf{p}_i \in \text{conv} \cup O \quad (15)
\]

where \( z_i = 1 \) if \( \mathbf{p}_i \in \text{conv} \), otherwise, \( z_i = -1 \). And \( w \in \mathbb{R}^2 \) and \( b \in \mathbb{R} \) are the parameters of the hyperplanes \( H_- \) and \( H_+ \), as shown in Fig. 4. The inequality constraint ensures that the obstacle and the agent system are positioned on the opposite side of \( H_- \) and \( H_+ \), respectively. The margin \( d \), also known as the distance between \( H_- \) and \( H_+ \), can be calculated by \( d = \frac{2}{\|w\|} \). To maximize \( d \), the optimization target is to
minimize $\|w\|$. Then, by the Lagrangian multiplier method, we can get

$$L(w, b, \lambda) = \frac{1}{2}\|w\|^2 + \sum_{p_i \in \text{conv} \cup O} \lambda_i \left(1 - z_i \left(w^T p_i + b\right)\right)$$

(16)

According to the KKT (Karush-Kuhn-Tucker) condition, the Hyperplane $H_-$ can be represented as

$$\sum_{p_i \in \text{conv} \cup O} \lambda_i z_i p_i^T p + 1 = 0$$

(17)

By solving the quadratic programming problem (15), the avoidance term is designed as follows:

$$F_o[k] = \sum_{h_i \in \text{conv}} F_h(h_i)$$

(18)

where

$$F_h(h_i) = \begin{cases} 1 + \cos \left(\pi \frac{d(h_i)}{d_{\text{min}}}\right) & d(h_i) < d_{\text{min}} \\ 0 & \text{else} \end{cases}$$

(19)

where $d_{\text{min}} > 0$ is the minimum safe margin. And $d(h_i)$ represents the distance between $h_i$ and the hyperplane $H_-$, which is calculated by

$$d(h_i) = \frac{(w^T h_i + b + 1)/\|w\|}{\min}$$

(20)

The bump function $F_h(h_i)$ is a repulsive potential function in the artificial potential field used in [22]. This function specifically ensures that the repulsive potential is active only when the distance $d(h_i)$ is smaller than the safety margin $d_{\text{min}}$. Also, to ensure this soft constraint is not violated, the weight $\alpha_o$ needs to be large enough.

**Constraints:** The constraints (7b) are the limitations of velocity and angular velocity. $v_i$ may have a positive or negative value, indicating forward or backward movement respectively. $\omega_i$ also can be negative or positive, indicating a turn to the left or right, respectively. The constraints (7c) and (7d) make sure the flexible mobile agent system moves in the rectangular bounded environment. Inequality (7e) expresses the distance limits between two agents due to the length of the tube $L$. It is assumed that $d_{\text{min}} = 0.4L$ and $d_{\text{max}} = 0.9L$.

### B. Transport Problem Formulation

After finishing enveloping the object, the next stage is to transport the object to the target receptacle. During this process, the controlled plant is the whole transport system instead of the mobile agent. The new algorithm framework is shown in Fig. 5. The corresponding optimization problem of this process is constrained by (i) dynamics (1) and (6), (ii)–(iv) and (vi) from the enveloping problem, and (vii) the new pulling constraint. The optimal tasks are the distance error and the control input. This optimization problem is built as

$$\min_{\mathbf{u}} \tilde{F} = \sum_{k=1}^{n} \left(\tilde{\alpha}_u \tilde{F}_u[k] + \tilde{\alpha}_d \tilde{F}_d[k] + \tilde{o} \tilde{F}_o[k]\right)$$

(21a)

subject to

$$\mathbf{u}_{\text{min}} \leq \mathbf{u}[k] \leq \mathbf{u}_{\text{max}}$$

(21b)

$$\chi_{\text{min}} \leq \chi[k] \leq \chi_{\text{max}}$$

(21c)

$$d_{\text{min}} \leq \left\|X_{1} - X_{2}\right\| \leq d_{\text{max}}$$

(21d)

$$\tilde{p}_{\text{min}} \leq \tilde{p}_t[k] \leq \tilde{p}_{\text{max}}$$

(21e)

$$y_t[k + 1] \geq 0$$

(21f)

where $\tilde{\alpha}_u$, $\tilde{\alpha}_d$, $\tilde{o}$ are the constant weights of each term. Different from the enveloping process, this objective function contains three parts. The control input term $\tilde{F}_u$ is the same as (8). The distance error between the transported object $\tilde{p}_t$ and the target zone $T_z$ is represented by

$$\tilde{F}_d[k] = (p_t[k] - T_z)^T \tilde{K}_d (p_t[k] - T_z)$$

(22)

where $\tilde{K}_d \in \mathbb{R}^{2 \times 2}$ is the positive definite matrix.

As shown in the left subfigure of Fig. 6, considering the object may occlude the tube in the view of the camera, the detection of feature points of the tube might be challenging. Also, due to the pulling force applied, the deformable tube between the object and the end robot will stretch and morph into a nearly straight shape. Then, the entire system is considered as a flexible triangle with the two end-robots and the object being transported forming the three vertices of it. So, for the obstacle avoidance term, the feature points $q_i$ are replaced by the center point $p_t$ of the transposed object, the new convex polygon of this system is $\text{conv}_{i} = \{X_1, X_2, X_3\}$ and the corresponding obstacle avoidance term is:

$$\tilde{F}_o[k] = \sum_{h_i \in \text{conv}_{i}} F_h(h_i)$$

(23)

Taking into account the distance between the center point of the transported object and its boundary, the safety margin $d_{\text{min}}$ needs to be bigger compared to the enveloping process.

**Constraints:** The constraints (21b)–(21d) are the same as the enveloping problem. The constraint (21e) is to make sure that the transported object also moves in the rectangular bounded...
environment. The inequality (21f) expresses the motion constraint of the pulling manipulation. The body frame $\sigma_b$ is shown as in the right subfigure of Fig. 6, where the origin $O_b$ is defined as the center of the transported object $p_t$. The direction of the $X_b$ axis is determined by the two end mobile robots, which is parallel to the vector $x_1 - x_2$. The predicted next step position $p_t^b[k+1]$ of the transported object under the body frame $\sigma_b[k]$ is calculated by

$$p_t^b[k+1] = \begin{bmatrix} \cos\phi[k] & \sin\phi[k] \\ -\sin\phi[k] & \cos\phi[k] \end{bmatrix} (p_t[k+1] - p_t[k]) \quad (24)$$

where $p_t^b[k+1] = [x_t^b[k+1], y_t^b[k+1]]^T \in \mathbb{R}^2$, $\phi[k]$ is the angle of the vector $x_1 - x_2$ in the world frame at the $k$-th step and calculated by

$$\phi[k] = \arctan(y_1[k] - y_2[k], x_1[k] - x_2[k]) \quad (25)$$

The non-negative constraint of $p_t^b[k+1]$ on $Y_\sigma[k]$ direction implies that the transported object will not move backward, which ensures the pulling constraint. Through the inequality (21f), the system can realize the forward pulling and the rotating in place with the transported object as the center, which shows the flexibility of our system.

**IV. RESULTS**

**A. Setup**

As shown in Fig. 7, we conducted this experimental study on a bounded rectangular platform with a size of 140 cm \times 80 cm. The flexible mobile agent system consists of two Mona robots [23] and one 30 cm-long deformable tube. By the connected component, the end of the tube is attached to the end robot but can rotate flexibly, making the shape of the tube relatively fixed and suitable for transport. The top-view camera obtains the state feedback information of the flexible mobile agent system and the manipulated object. To get the shape information of the tube, we add some red markers evenly on the tube as the feature points and use the OpenCV libraries to process the images and track the feature points. The receptacle zone is designed as a rectangle zone at one corner of the platform. The Mona robots’ linear velocity and angular velocity are regulated wirelessly by a host PC. In our experiments, the time step of MPC is set as $0.15$ s. The prediction horizon $N_s$ is 30, and the control step $N_h$ is 4. The max velocity of the robots is designed as $0.15$ m/s, and the max angle velocity is 0.4 rad/s.

We solve the proposed nonlinear constrained optimization problems (7) and (21) by Interior Point OPTimizer (IPOPT) solver [24] which is a software library for large-scale nonlinear optimization of continuous systems. Due to the control step $N_s = 4$, the predicted control inputs of $5$-th step and beyond can be set as the initial guesses of the next optimization. The initial values of control inputs without the predicted value are set to zero. Videos of the experiments can be found at https://vimeo.com/917007028.

**B. Real-World Experiments**

1) Experiments without Obstacles: First, the proposed algorithm is evaluated by conducting tests using objects of various shapes placed at different locations in an environment without any obstacles. Fig. 8 illustrates the process of envelopment and transportation of three different cases. The light snapshots are the states during this process, while the dark snapshot is the final configuration of each task. The blue rectangle of the top left corner is the target receptacle. Objects with diverse shapes, including both convex and nonconvex shapes, are tested to demonstrate the adaptability of our proposed system.

The quantitative experiment results are given in Fig. 9. The superscript “$\star$” presents the corresponding predicted variable. The steer term $F_\theta$ is activated when the distance error is below 0.35 m, which means the system initiates a rotation towards the object, aligning itself to face the object with the open orientation. When the direction error $\|p_t^\star - p_t\|$ and the relative orientation $\theta$ are both close enough to zero, the enveloping task is finished. The Least square method is used to fit the dynamic model (2) and to estimate the Jacobian matrix $J(x)$, based on the sampled data of position changes of the feature points and end robots during the sample time $\Delta_t$: $\Delta q = q(t + \Delta t) - q(t)$ and $\Delta x = x(t + \Delta t) - x(t)$. As shown in the middle left figure in Fig. 9, the error $\sum ||q_t^\star - q_t||$, that presents the error in predicting the position of the tube, can converge to within 5 cm bound, which is acceptable for our task. Likewise, the matrix $J_t$ is also updated during the transportation process using the Least Square technique. The position prediction errors of the transported object in both $X$ and $Y$ directions are depicted in the figures located at the bottom, exhibiting values close to...
Fig. 9. Quantitative results of errors during experiments in Fig. 8. The top left figure is the evolution of the distance error during the enveloping process. The top right figure is the evolution of $\theta$ in steer term during the enveloping process. The middle left figure is the sum of errors between the real feature points and the predicted value by model (2). The middle right figure is the distance error of the transported object during the transport task. The bottom figures are the error between the predicted value of the position of the object and the real position in both the $x$ and $y$ axis, respectively.

Remark. The model error of the tube and transported objects can not converge to zero strictly in experiments. The models are not fixed and are affected by the relative configuration of the system, which is not strictly quasi-static throughout the motion process. However, the error of the proposed estimated model is acceptable in our task.

2) Experiments with Obstacles: To test the performance of obstacle avoidance, a series of manipulation tasks with obstacles of different shapes are conducted. Fig. 10 presents four sets of experimental results. A snake-shaped obstacle (Fig. 10(a) and Fig. 10(b)) and a rectangular obstacle (Fig. 10(c) and Fig. 10(d)) are used to represent the non-convex and convex obstacle, respectively. The performed experiments involve obstacle avoidance on the lateral sides of the two end robots (Fig. 10(a), Fig. 10(b)) and on the side of the tube (Fig. 10(c), Fig. 10(d)) during the enveloping task. Fig. 10(b) also shows the avoidance of obstacles during the transportation process. As shown in Fig. 10(e)–Fig. 10(f), two optimization targets of the enveloping problem, the distance error $\|p^f - p_l\|$ and the relative orientation $\theta$, have both converged to zero. Also, the objects being transported are positioned within the designated area with permissible deviations. In conclusion, our system has successfully completed the task of envelopment and transport with obstacles of varying shapes and target objects positioned differently.

Remark. Since the error of the estimated model exists, chattering occurs, particularly when the tube side is maneuvering around obstacles, as shown in the c and d curves of Fig. 10(e). The obstacle avoidance processes take a longer time but do not lead to the failure of the task.

Fig. 10. Snapshots of experiments of the proposed method applied in different obstacle situations (a)–(d) and the corresponding quantitative results (e)–(g).

3) Comparison: For obstacle avoidance during the enveloping process, the comparison of our maximum margin method and the classic Artificial Potential Field (APF) is conducted, and the results are given in Table I and II of Fig. 11. In APF, the repulsive potential fields in [25] of each feature point $q_j$ and the two agent robots $x_i$ are considered and added to the objective function (7a). The comparison experiments are conducted using rectangular obstacles of varying sizes, made up of different numbers of cubes. Failure cases of APF in Fig. 11 show the riskiness of enveloping the obstacle, particularly when dealing with a small obstacle. It is primarily due to the conflicting repulsive potential fields that exist within different components of the system with a linear shape. In contrast, our proposed approach effectively avoids this potential hazard. Moreover, Ours has shorter calculation times, which is more significant when dealing with a robot with more feature points.

C. Failure Case

The enveloping algorithm involves two distinct targets, namely the position and the direction. It is worth noting that the presence of these targets may potentially lead to collision. As shown in Fig. 12(a), the initial orientation of the flexible mobile agent system is opposite from the target configuration...
and there is limited room for rotating, then, the enveloping task failed. The two terms $F_d$ and $F_d$ cannot be simultaneously reduced, resulting in the optimization problem being trapped in a local optimum. Fig. 12(b) is an ideal situation in which the system moves away from the local optimum and discovers the global optimization. In scenarios where the system possesses a more favorable initial direction configuration (with a smaller $\theta$) or has a larger space for maneuvering (with a larger position error), as shown in Fig. 12(c) and Fig. 12(d), correspondingly, the likelihood of encountering a local minimum is significantly reduced.

V. Conclusion

In this paper, the problem of nonprehensile object manipulation by the proposed flexible mobile agent system has been addressed. The proposed system consists of two mobile end robots linked by a deformable tube, which has the potential for object manipulation by the deformable material. The corresponding local planning and control frameworks of the proposed new system for object enveloping and transport are presented and successfully tested with real experiments. The position and orientation requirements and obstacle avoidance, nonprehensile manipulation are all formulated and integrated into the optimization problem.

Our work shows the possibilities and challenges associated with nonprehensile manipulation using the combination of deformable materials and mobile robots. Furthermore, this system can be extended to multi-object nonprehensile manipulation, which can better utilize the flexibility of this system.

Fig. 12. Snapshots of the experimental enveloping process and the corresponding quantitative results. (a) failed, the other three succeeded.