Robust Consensus of a Class of Perturbed Port-Hamiltonian Systems With Time Delays

Jose Guadalupe Romero, *Member, IEEE*, Alejandro Donaire, *Member, IEEE*, Emmanuel Nuño, *Senior Member, IEEE*, and David Navarro-Alarcon, *Senior Member, IEEE*

***Abstract*—In this article, we present a distributed control system design for the consensus of nonlinear multiagent systems subject to non-ideal communication channels and external disturbances. The agents are modeled as a class of perturbed port-Hamiltonian systems—a formalism to model complex physical and engineering systems—and the imperfection in the communication channels is represented by time-delays that are assumed to be bounded. We show that the proposed distributed controller ensures global asymptomatic convergence of the agents to a consensus equilibrium, despite external disturbances and time-delays. We also extend the results to Euler–Lagrange agents as a corollary of our main result. Simulation results, for a network of five, 2-degrees-of-freedom robotic manipulators show the performance of our proposed control design.**

***Index Terms*—Consensus, disturbance rejection, PID controller, port-Hamiltonian (pH) systems.**

I. Introduction

The fundamental results of [1] and [2], on the consensus control of multiagent dynamical systems, paved the way to a plethora of consensus controller designs. Consensus means that the state variables of the agents converge to a common agreement value. The dynamical description of a multiagent system contains a twofold: the dynamics of the agents (nodes) and their interconnection topology. Besides the nonlinearities of the dynamical agents, the consensus control becomes more complex when communication delays and input disturbances arise [3]. Input disturbances are a natural effect present in many physical systems, and they can be caused by sensor measurements, system noise, external forces, or biases on the control input. The presence of these disturbances can generate a set of equilibria, if we consider a single agent, such that the desired equilibrium point might not be reached [4]. Consequently, in the multiagent case, the natural consensus equilibrium is shifted. In addition, if delays are considered, instability or other deleterious effects might appear [5].

Several results have been obtained to ensure consensus for multiple Euler–Lagrange (EL) systems [6]. The beginnings of these developments can be traced back to the works in [7] and [8], where neither delays nor external disturbances have been considered. Later in [9] and [10], and adaptive controller and a Proportional plus damping (P+d) controller, respectively, have been designed to deal with communication delays. Recently, using the internal model approach of [11], the P+d scheme has been extended to deal with external disturbances in [12]. However, the result in [12] requires the internal model of the disturbance, which might be unrealistic in some practical applications and it cannot reject constant disturbances. Another work that deals with external disturbances is [13] that reports a solution to the leader–follower robust synchronization of uncertain EL systems but that it requires the exact knowledge of the delays and it cannot be generalized to the leaderless consensus scenario. An adaptive neural network has been employed in [14] to solve the leader–follower consensus when the information is sent using an event-triggered mechanism when delays are not considered. The sliding-modes control technique has also been employed to deal with parameter uncertainty and with external perturbations [15], [16], [17]. However, these solutions are prone to the undesired effect of chattering [18]. A different framework to deal with disturbances is to employ PID controllers, as in [19], or disturbance observers, as in [20]. However, these results have been designed for vehicles that exhibit nonholonomic constraints and cannot be applied to agents with a more general dynamic description, as those in our present work.

Consensus of multiple agents exploiting the passivity property—intrinsic to many physical systems—has also been reported in several works. For example, in [21] and [22], consensus algorithms have been employed to solve distributed optimization problems and to solve the formation problem for double integrator dynamics and power systems, respectively. Most recently, passivity has also been used to establish the formation of nonhomonomic mobile robots [23] and the consensus of single-input-single-output systems interconnected by balanced digraphs [24]. The first work ensuring consensus of EL systems via passivity-based control and without velocity measurement is presented in [25]. None of these works have considered external disturbances.

Although the consensus of nonlinear systems, including port-Hamiltonian (pH) systems, has been studied in various works, for example, [26] and [27], none of them has found a solution to the consensus problem in the presence of external disturbances and delayed communications. Moreover, several of the proposed solutions mentioned in the above literature relies on the exact knowledge of the model. In this work, we are interested in modeling the nodes as dynamical systems in the pH form. It is worth mentioning that this class of systems is general as they can model different physical phenomena: mechanical, electrical, hydraulic, thermal, chemical, biological, etc. [28], [29].

Received 20 June 2025; accepted 13 July 2025. Date of publication 28 July 2025; date of current version 5 December 2025. This work was supported by the Australian Government through the Australian Research Council's Discovery Projects funding scheme under Project DP220103637. Recommended by Associate Editor Z. Kan. *(Corresponding author: Emmanuel Nuño.)*

Jose Guadalupe Romero is with the Departamento Académico de Ingeniería Eléctrica y Electrónica, Instituto Tecnológico Autónomo de México, Mexico City 01080, México (e-mail: jose.romerovelazquez@itam.mx).

Alejandro Donaire is with the School of Engineering, The University of Newcastle, Callaghan, NSW 2308, Australia (e-mail: alejandro.donaire@newcastle.edu.au).

Emmanuel Nuño is with the Department of Computer Science, University of Guadalajara, Guadalajara, Jalisco 44430, Mexico (e-mail: emmanuel.nuno@academicos.udg.mx).

David Navarro-Alarcon is with the Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom FG 638, Hong Kong (e-mail: dna@ieee.org).

Digital Object Identifier 10.1109/TAC.2025.3593232

In this article, we first focus on solving the consensus problem using decentralized controllers for a class of pH systems subject to constant input disturbances. We assume that the pH agents transmit information through a network topology that is modeled with a static, connected, and undirected graph. The interconnection generates variable time delays that are assumed bound. Furthermore, our main contribution is a novel decentralized controller that has the structure of a P+d injection scheme with an innovative integral term (PID) that is designed using the model of the system, and whose physical parameters are assumed known. Using a Lyapunov–Krasovskii functional, we provide a sufficient condition on the controller's integral gain to ensure consensus to an agreement point. We also prove that our proposal solves the open problem of consensus and disturbance rejection for multiple perturbed EL systems with time delays. This can be done, thanks to the fact that EL systems and pH systems are related through the Legendre transformation, as will become clear in Section V. These two formalisms provide two different modeling frameworks with different properties [30]. Finally, we provide numerical simulations that show the performance of the proposed control system.

II. Port-Hamiltonian Model

In this note, we are interested in solving the problem of *robust* consensus of N pH systems that exhibit external disturbances. The dynamics of each ith agent is described in the pH form as follows:

$$\dot{\mathbf{x}}_i = \mathcal{J}_i(\mathbf{x}_i)\nabla H_i(\mathbf{x}_i) + \mathcal{G}(\mathbf{x}_i)(\mathbf{u}_i + \mathbf{d}_i) \tag{1}$$

where $\mathbf{x}_i = [\mathbf{x}_{1i}^\top \ \ \mathbf{x}_{2i}^\top]^\top$, with $\mathbf{x}_{1i}, \mathbf{x}_{2i} \in \mathbb{R}^n$, is the state, $\mathbf{u}_i \in \mathbb{R}^n$ is the control input, and $\mathbf{d}_i \in \mathbb{R}^n$ is the external unknown disturbance that can be constant *or* piecewise constant. The Hamiltonian $H_i : \mathbb{R}^{2n} \to \mathbb{R}$ is a function that represents the total energy of the system, $\mathcal{J}_i(\mathbf{x}_i) \in \mathbb{R}^{2n\times 2n}$ is the interconnection matrix, which verifies the property $\mathbf{x}_i^\top \mathcal{J}_i(\mathbf{x}_i)\mathbf{x}_i = 0$ and $\mathcal{G}_i(\mathbf{x}_i) \in \mathbb{R}^{2n\times n}$ is the input matrix. We now characterize the following class of systems to be addressed in this note.

1) The interconnection matrix has the form

$$\mathcal{J}_i(\mathbf{x}_i) = \begin{bmatrix} 0_{n\times n} & \Phi_i(\mathbf{x}_{1i}) \\ -\Phi_i^\top(\mathbf{x}_{1i}) & J_i(\mathbf{x}_{1i}, \mathbf{x}_{2i}) \end{bmatrix} \tag{2}$$

and $\Phi_i(\mathbf{x}_{1i}) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ and $J(\mathbf{x}_{1i}, \mathbf{x}_{2i}) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ being nonsingular matrices.

2) The input matrix can be described as follows:

$$\mathcal{G}_i(\mathbf{x}_i) = [0_n, \ \Omega_i(\mathbf{x}_{1i})] \tag{3}$$

with positive definite and symmetric matrix $\Omega_i(\mathbf{x}_{1i}) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, for all $\mathbf{x}_{1i} \in \mathbb{R}^n$.

In addition, we assume the following.

A1 Matrices $\Phi_i(\mathbf{x}_{1i}), \Omega_i(\mathbf{x}_{1i})$ are bounded for all $\mathbf{x}_{1i} \in \mathbb{R}^n$ and there exists a positive number $c_{Ji} > 0$ such that $|J(\mathbf{x}_{1i}, \mathbf{x}_{2i})| \leq c_{Ji}|\mathbf{x}_{2i}|$.

The Hamiltonian is given by

$$H_i(\mathbf{x}_i) = \frac{1}{2}\mathbf{x}_{2i}^\top \mathcal{M}_i^{-1}\mathbf{x}_{2i} + \mathcal{P}(\mathbf{x}_{1i})$$

where $\mathcal{M}_i : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is a constant symmetric positive matrix and $\mathcal{P}(\mathbf{x}_{1i}) : \mathbb{R}^n \to \mathbb{R}$ is the potential energy function.

The systems verifying (2), (3), and assumption **A1** include, but are not limited to, marine vehicles [31], [32], satellites [33], spacecraft [34], fully actuated hexarotors [35], mechanical systems interacting with unknown elastic environments [36], and physical system modeled in EL form. The solution for EL systems will be discussed in Section V.

The consensus objective is to drive $\mathbf{x}_{1i}$ to an agreement value $\mathbf{x}_c \in \mathbb{R}^n$ that is found through a distributed consensus algorithm, while ensuring that $\mathbf{x}_{2i}$ converges to zero. Formally, the control objective is the following.

Leaderless Consensus Objective: Design a distributed *PID controller* ensuring that all pH systems globally and asymptotically converge to a consensus point, that is

$$\lim_{t\to\infty} \mathbf{x}_{1i}(t) = \mathbf{x}_c; \quad \lim_{t\to\infty} \mathbf{x}_{2i}(t) = 0 \tag{4}$$

for some $\mathbf{x}_c \in \mathbb{R}^n$, while rejecting the external disturbances. ◁

Remark 1: If the application requires that the pH systems converge to a predefined position in a given formation, then we can define the variable $\bar{\mathbf{x}}_{1i} := \mathbf{x}_{1i} - \boldsymbol{\delta}_i$, where $\boldsymbol{\delta}_i$ is the desired (constant) position of the ith-pH system relative to the center of a given *desired formation* pattern. Then, we can ensure that all the pH reach their given relative position while agreeing on the center of the formation pattern. In such a case, the control objective is reformulated to ensure that

$$\lim_{t\to\infty} \bar{\mathbf{x}}_{1i}(t) = \mathbf{x}_c; \quad \lim_{t\to\infty} \mathbf{x}_{2i}(t) = 0. \tag{5}$$

◁

III. Disturbance Rejection for a Single PH System

For a more comprehensible understanding of our control design, in this section, we first show the disturbance rejection result for a single pH system. The objective is to ensure that the position of a pH system converges to a given constant desired position, defined as $\mathbf{x}_1^\star \in \mathbb{R}^n$. In this case, (1) takes the following form:

$$\dot{\mathbf{x}} = \mathcal{J}(\mathbf{x})\nabla H + \mathcal{G}(\mathbf{x})(\mathbf{u} + \mathbf{d}). \tag{6}$$

The controller is composed of a potential energy cancellation term plus a PID controller and it is given by

$$\mathbf{u} = \Omega^{-1}(\mathbf{x}_1)\Phi^\top(\mathbf{x}_1)\left[\nabla\mathcal{P} - k_p\tilde{\mathbf{x}}_1\right] - k_d\mathcal{M}^{-1}\mathbf{x}_2 - k_I\mathcal{M}^{-1}\boldsymbol{\xi} \tag{7}$$

where $\tilde{\mathbf{x}}_1$ is the position error defined as $\tilde{\mathbf{x}}_1 := \mathbf{x}_1 - \mathbf{x}_1^\star$ and $\boldsymbol{\xi} \in \mathbb{R}^n$ has the following dynamics:

$$\dot{\boldsymbol{\xi}} = k_p\Phi^\top(\mathbf{x}_1)\tilde{\mathbf{x}}_1 + \left[k_r\Omega(\mathbf{x}_1) - J(\mathbf{x}_1, \mathbf{x}_2)\right]\mathcal{M}^{-1}\mathbf{x}_2 \tag{8}$$

where $k_r > 0$ and $k_p, k_I, k_d > 0$ are the PID control gains.

The closed-loop system is then given by

$$\begin{aligned}
\dot{\tilde{\mathbf{x}}}_1 &= \Phi(\mathbf{x}_1)\mathcal{M}^{-1}\mathbf{x}_2 \\
\dot{\mathbf{x}}_2 &= -k_p\Phi^\top(\mathbf{x}_1)\tilde{\mathbf{x}}_1 - \Omega(\mathbf{x}_1)\mathcal{M}^{-1}\left[k_d\mathbf{x}_2 + k_I\tilde{\boldsymbol{\xi}}\right] \\
&\quad + J(\mathbf{x}_1, \mathbf{x}_2)\mathcal{M}^{-1}\mathbf{x}_2 \\
\dot{\tilde{\boldsymbol{\xi}}} &= k_p\Phi^\top(\mathbf{x}_1)\tilde{\mathbf{x}}_1 + k_r\Omega(\mathbf{x}_1)\mathcal{M}^{-1}\mathbf{x}_2 - J(\mathbf{x}_1, \mathbf{x}_2)\mathcal{M}^{-1}\mathbf{x}_2
\end{aligned} \tag{9}$$

where we have defined $\tilde{\boldsymbol{\xi}} := \boldsymbol{\xi} - \frac{1}{k_I}\mathcal{M}\mathbf{d}$.

Proposition 1: Consider a single pH system of the form (6), controlled with (7) and (8). Then, the equilibrium point

$$(\tilde{\mathbf{x}}_1, \mathbf{x}_2, \tilde{\boldsymbol{\xi}}) = (\mathbf{0}, \mathbf{0}, \mathbf{0})$$

is globally asymptotically stable, provided that $k_r = 5k_I$ and $k_d = 3k_I$. ◁

Proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2}\mathbf{x}_2^\top\mathcal{M}^{-1}\mathbf{x}_2 + \frac{1}{2}(\mathbf{x}_2 + \tilde{\boldsymbol{\xi}})^\top\mathcal{M}^{-1}(\mathbf{x}_2 + \tilde{\boldsymbol{\xi}}) + \frac{1}{2}k_p|\tilde{\mathbf{x}}_1|^2$$

which is positive definite and radially unbounded. Its time-derivative is

$$\dot{V} = \mathbf{x}_2^\top\mathcal{M}^{-1}\dot{\mathbf{x}}_2 + (\mathbf{x}_2 + \tilde{\boldsymbol{\xi}})^\top\mathcal{M}^{-1}(\dot{\mathbf{x}}_2 + \dot{\tilde{\boldsymbol{\xi}}} + k_p\tilde{\mathbf{x}}_1^\top\dot{\tilde{\mathbf{x}}}_1.$$

Evaluating each element of $\dot{V}$ along (9) yields

$$\mathbf{x}_2^\top \mathcal{M}^{-1}\dot{\mathbf{x}}_2 = -k_p \mathbf{x}_2^\top \mathcal{M}^{-1}\Phi^\top(\mathbf{x}_1)\tilde{\mathbf{x}}_1 - \mathbf{x}_2^\top \mathcal{M}^{-1}\Omega(\mathbf{x}_1)\mathcal{M}^{-1}\left[k_d\mathbf{x}_2 + k_I\tilde{\boldsymbol{\xi}}\right]$$

where the term $\mathbf{x}_2^\top\mathcal{M}^{-1}J(\mathbf{x}_1,\mathbf{x}_2)\mathcal{M}^{-1}\mathbf{x}_2 = 0$ due to the skew-symmetric property of $J(\mathbf{x}_1,\mathbf{x}_2)$. Further,

$$\dot{\mathbf{x}}_2 + \dot{\tilde{\xi}} = -\Omega(\mathbf{x}_1)\mathcal{M}^{-1}\left[(k_d - k_r)\mathbf{x}_2 + k_I\tilde{\boldsymbol{\xi}}\right]$$

and $\tilde{\mathbf{x}}_1^\top\dot{\tilde{\mathbf{x}}}_1 = \tilde{\mathbf{x}}_1^\top\Phi(\mathbf{x}_1)\mathcal{M}^{-1}\mathbf{x}_2$.

After doing some algebraic manipulations and setting $k_r = 5k_I$ and $k_d = 3k_I$, yields

$$\dot{V} = -k_I\mathbf{x}_2^\top\mathcal{M}^{-1}\Omega(\mathbf{x}_1)\mathcal{M}^{-1}\mathbf{x}_2 - k_I\tilde{\boldsymbol{\xi}}^\top\mathcal{M}^{-1}\Omega(\mathbf{x}_1)\mathcal{M}^{-1}\tilde{\boldsymbol{\xi}}.$$

Since $\mathcal{M}^{-1}$ and $\Omega(\mathbf{x}_1)$ are symmetric and positive definite matrices, respectively, for all $\mathbf{x}_1 \in \mathbb{R}^n$, then there exists a positive constant λ such that $\dot{V} \le -k_I\lambda(|\mathbf{x}_2|^2 + |\tilde{\boldsymbol{\xi}}|^2)$. Hence, $\dot{V}$ is negative semidefinite and it only vanishes when $\mathbf{x}_2 = \mathbf{0}$ and $\tilde{\boldsymbol{\xi}} = \mathbf{0}$. Finally, the proof is finished applying Barbashin–Krasovskii–LaSalle's Invariance Theorem [37]. □

IV. Main Result

This section presents the main result of this work, which is the solution to the consensus problem, as described in Section II.

We model the complete system using graph theory, where each pH system is a node and the vertices of the graph are established through communication links. Therefore, the interconnection of the pH systems is modeled using the Laplacian matrix $L := [\ell_{ij}] \in \mathbb{R}^{N\times N}$, whose elements are defined as

$$\ell_{ij} = \begin{cases} \sum_{k\in\mathcal{N}_i} a_{ik}, & i = k \\ -a_{ik}, & i \ne k \end{cases} \tag{10}$$

where $i \in \bar{N}$ and $\mathcal{N}_i$ is the set of transmitting information to the ith system, $a_{ik} > 0$ if $k \in \mathcal{N}_i$ and $a_{ik} = 0$, otherwise.

Regarding the graph topology, we have the following assumption.

A2 The interconnection graph is *undirected, static, and connected.*

Remark 2: Assumption **A2** is ubiquitous in passivity-based controllers [6], [25]. This is due to the following facts for the Laplacian matrix: 1) it has a zero row sum, i.e., $L1_N = 0_N$; 2) it is symmetric; 3) it has a single zero-eigenvalue and the rest of its spectrum is strictly positive; and 4) the only vector living in the kernel of L is span$\{1_N\}$, where 1_N is a column vector of ones of size N. ◁

Now, since communication links are required to transmit information from one agent to another, transmission delays are unavoidable [38], [39]. Hence, the following assumption holds.

A3 The communication from the jth to the ith pH system is subject to a variable time-delay denoted as $T_{ji}(t)$ that is bounded by a known upper bound $\overline{T}_{ji} \ge 0$ and has bounded time derivatives.

In order to establish the leaderless consensus objective, the ith-agent receives the values $\mathbf{x}_{1j}$ from their neighbors $j \in \mathcal{N}_i$ through a communication channel satisfying **A2** and **A3**. We should remark that **A3** can be relaxed to deal with nondifferentiable delays by constructing a strict Lyapunov–Krasovskii functional, as in [40]. This is, however, an open problem that is not treated here.

Using the incoming values, let us define the error as

$$\mathbf{e}_i := \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{x}_{1i} - \mathbf{x}_{1j}(t - T_{ji}(t))\right). \tag{11}$$

Then, if we can prove that $\mathbf{e}_i$ and $\mathbf{x}_{2i}$ globally converge to zero the desired objective is satisfied. To see this, note that the error can be written as

$$\mathbf{e}_i = \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{x}_{1i} - \mathbf{x}_{1j} + \int_{t-T_{ji}(t)}^{t}\dot{\mathbf{x}}_{1j}(\sigma)d\sigma\right). \tag{12}$$

Pilling-up the N positions and errors as $\mathbf{x}_1 := [\mathbf{x}_{11}^\top,\ldots,\mathbf{x}_{1^\cdot N}^\top]^\top$ and $\mathbf{e} := [\mathbf{e}_1^\top,\ldots,\mathbf{e}_N^\top]^\top$, correspondingly, then it holds that

$$\mathbf{e} = (L\otimes I_n)\mathbf{x}_1 + \mathrm{col}\left[\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{x}}_{1j}(\sigma)d\sigma\right].$$

If $\mathbf{x}_{2i}$ converges to zero, then using (9), then it ensures that $\dot{\mathbf{x}}_{1j}$ also converges to zero, and hence, the following implication holds:

$$\lim_{t\to\infty}\mathbf{x}_{2i}(t) = \mathbf{0} \Rightarrow \lim_{t\to\infty}\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{x}}_{1j}(\sigma)d\sigma = \mathbf{0}.$$

Therefore, due to the properties of the Laplacian matrix, one has that there exists $\mathbf{x}_c \in \mathbb{R}^n$ such that

$$\lim_{t\to\infty}\mathbf{e}(t) = \mathbf{0} \cap \lim_{t\to\infty}\mathbf{x}_{2i}(t) = \mathbf{0} \Rightarrow \lim_{t\to\infty}\mathbf{x}_1(t) = 1_N\otimes\mathbf{x}_c$$

where $\otimes$ is the standard Kronecker product and I_n is the identity matrix of size $n\times n$.

We design the controller following the insights of the previous section with four different terms, one that cancels out the potential energy contribution to the dynamics, plus the proportional, the damping injection, and the disturbance rejection terms. So, our proposed scheme is given by

$$\begin{aligned}\mathbf{u}_i &= \Omega_i^{-1}(\mathbf{x}_{1i})\Phi_i^\top(\mathbf{x}_{1i})\left[\nabla\mathcal{P}_i(\mathbf{x}_{1i}) - k_{pi}\mathbf{e}_i\right] - k_{di}\mathcal{M}_i^{-1}\mathbf{x}_{2i} \\ &\quad - k_{Ii}\mathcal{M}_i^{-1}\boldsymbol{\xi}_i \\ \dot{\boldsymbol{\xi}}_i &= k_{pi}\Phi_i^\top(\mathbf{x}_{1i})\mathbf{e}_i + \left[k_{ri}\Omega_i(\mathbf{x}_{1i}) - J_i(\mathbf{x}_{1i},\mathbf{x}_{2i})\right]\mathcal{M}_i^{-1}\mathbf{x}_{2i}\end{aligned} \tag{13}$$

where $k_{ri} > 0$, and $k_{pi}, k_{di}, k_{Ii} > 0$ are the proportional, the damping injection, and the integral gains, respectively. Note that the structure of the controller resembles the PID controller designed in the previous section; however, strictly speaking, it is not because we do not employ the derivative of the error $\dot{\mathbf{e}}_i$.

The resulting closed-loop system for each pH agent is

$$\begin{aligned}\dot{\mathbf{x}}_{1i} &= \Phi_i(\mathbf{x}_{1i})\mathcal{M}_i^{-1}\mathbf{x}_{2i} \\ \dot{\mathbf{x}}_{2i} &= -k_{pi}\Phi_i^\top(\mathbf{x}_{1i})\mathbf{e}_i - \Omega_i(\mathbf{x}_{1i})\mathcal{M}_i^{-1}\left[k_{di}\mathbf{x}_{2i} + k_{Ii}\tilde{\boldsymbol{\xi}}_i\right] \\ &\quad + J_i(\mathbf{x}_{1i},\mathbf{x}_{2i})\mathcal{M}_i^{-1}\mathbf{x}_{2i} \\ \dot{\tilde{\boldsymbol{\xi}}}_i &= k_{pi}\Phi_i^\top(\mathbf{x}_{1i})\mathbf{e}_i + k_{ri}\Omega_i(\mathbf{x}_{1i})\mathcal{M}_i^{-1}\mathbf{x}_{2i} - J_i(\mathbf{x}_{1i},\mathbf{x}_{2i})\mathcal{M}_i^{-1}\mathbf{x}_{2i}\end{aligned} \tag{14}$$

where the disturbance error $\tilde{\boldsymbol{\xi}}_i \in \mathbb{R}^n$ is $\tilde{\boldsymbol{\xi}}_i := \boldsymbol{\xi}_i - \frac{1}{k_{Ii}}\mathcal{M}_i\mathbf{d}_i$.

The main result of this article is the following.

Theorem 1: Consider the systems (1) controlled by (13). Suppose that Assumptions **A1–A3** hold. Set the gains as $k_{ri} = 5\,k_{Ii}$ and $k_{di} = 3\,k_{Ii}$. In addition, ensure that the following condition holds:

$$k_{Ii} > \frac{1}{2}\frac{k_{pi}}{\phi_i}\sum_{j\in\mathcal{N}_i} a_{ij}\left(c_i + \frac{\overline{T}_{ij}^2}{c_j}\right) \tag{15}$$

for all $i \in \bar{N}, j \in \mathcal{N}_i$, for some $c_i > 0$, and where $\phi_i := \min\{\mathrm{eig}\{\Phi_i^{-\top}(\mathbf{x}_{1i})\Omega_i(\mathbf{x}_{1i})\Phi_i^{-1}(\mathbf{x}_{1i})\}\}$. In these conditions, the desired consensus objective holds globally, i.e., (4) holds for any $\mathbf{x}_{1i}(0), \mathbf{x}_{2i}(0), \boldsymbol{\xi}_i(0) \in \mathbb{R}^n$. ◁

Proof: We start the proof by defining the Lyapunov candidate function whose time-derivative has negative terms with regards to $\mathbf{x}_{2i}$ and $\boldsymbol{\xi}_i$, that is

$$\mathcal{U}_i = \frac{1}{2}\mathbf{x}_{2i}^\top \mathcal{M}_i^{-1}\mathbf{x}_{2i} + \frac{1}{2}(\mathbf{x}_{2i} + \tilde{\boldsymbol{\xi}}_i)^\top \mathcal{M}_i^{-1}(\mathbf{x}_{2i} + \tilde{\boldsymbol{\xi}}_i).$$

Following the same procedure as in the proof of Proposition 1 and evaluating $\dot{\mathcal{U}}_i$ along (14) yields

$$\begin{aligned}\dot{\mathcal{U}}_i =& -k_{Ii}\mathbf{x}_{2i}^\top \mathcal{M}_i^{-1}\Omega_i(\mathbf{x}_{1i})\mathcal{M}_i^{-1}\mathbf{x}_{2i} - k_{pi}\mathbf{x}_{2i}^\top \mathcal{M}_i^{-1}\Phi_i^\top(\mathbf{x}_{1i})\mathbf{e}_i \\ & - k_{Ii}\tilde{\boldsymbol{\xi}}_i^\top \mathcal{M}_i^{-1}\Omega_i(\mathbf{x}_{1i})\mathcal{M}_i^{-1}\tilde{\boldsymbol{\xi}}_i \\ =& -k_{Ii}\dot{\mathbf{x}}_{1i}^\top \Phi_i^{-\top}(\mathbf{x}_{1i})\Omega_i(\mathbf{x}_{1i})\Phi_i^{-1}(\mathbf{x}_{1i})\dot{\mathbf{x}}_{1i}^\top - k_{pi}\dot{\mathbf{x}}_{1i}^\top \mathbf{e}_i \\ & - k_{Ii}\tilde{\boldsymbol{\xi}}_i^\top \mathcal{M}_i^{-1}\Omega_i(\mathbf{x}_{1i})\mathcal{M}_i^{-1}\tilde{\boldsymbol{\xi}}_i\end{aligned}$$

where to obtain the second equality we have employed the first equation of (14). Now, defining $\mu_i := \min\{\mathrm{eig}\{\mathcal{M}_i^{-1}\Omega_i(\mathbf{x}_{1i})\mathcal{M}_i^{-1}\}\} > 0$, it holds that

$$\dot{\mathcal{U}}_i \leq -k_{Ii}\phi_i|\dot{\mathbf{x}}_{1i}|^2 - k_{Ii}\mu_i|\tilde{\boldsymbol{\xi}}_i|^2 - k_{pi}\dot{\mathbf{x}}_{1i}^\top \mathbf{e}_i.$$

We now define a Lyapunov–Krasovskii functional (see, e.g., [40]) that dominates the effects of the delays in the communications and that is given by

$$\begin{aligned}\mathcal{W}_i =& \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}|\mathbf{x}_{1i} - \mathbf{x}_{1j}|^2 \\ & + \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}\frac{\overline{T}_{ji}}{c_i}\int_{-\overline{T}_{ji}}^{0}\int_{t+\sigma}^{t}|\dot{\mathbf{x}}_{1j}(\nu)|^2 d\nu d\sigma \end{aligned} \tag{16}$$

where $c_i > 0$ is a constant that will be defined later. The time derivative of (16) is

$$\begin{aligned}\dot{\mathcal{W}}_i =& \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_{1i} - \mathbf{x}_{1j})^\top(\dot{\mathbf{x}}_{1i} - \dot{\mathbf{x}}_{1j}) \\ & + \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}\frac{\overline{T}_{ji}}{c_i}\left[\overline{T}_{ji}|\dot{\mathbf{x}}_{1j}|^2 - \int_{t-\overline{T}_{ji}}^{t}|\dot{\mathbf{x}}_{1j}(\sigma)|^2 d\sigma\right].\end{aligned}$$

At this point, we want to underline that

$$\begin{aligned}\dot{\mathbf{x}}_{1i}^\top \mathbf{e}_i + \dot{\mathcal{W}}_i =& -\frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_{1i} - \mathbf{x}_{1j})^\top(\dot{\mathbf{x}}_{1i} + \dot{\mathbf{x}}_{1j}) \\ & - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{x}}_{1i}^\top \int_{t-T_{ji}(t)}^{t}\dot{\mathbf{x}}_{1j}(\sigma)d\sigma \\ & + \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}\frac{\overline{T}_{ji}}{c_i}\left[\overline{T}_{ji}|\dot{\mathbf{x}}_{1j}|^2 - \int_{t-\overline{T}_{ji}}^{t}|\dot{\mathbf{x}}_{1j}(\sigma)|^2 d\sigma\right].\end{aligned}$$

On the one hand, using Young's and Cauchy–Schwartz's inequalities (see, e.g., [41]) we have that, for any $c_i > 0$, the following inequality holds:

$$\begin{aligned}-\dot{\mathbf{x}}_{1i}^\top \int_{t-T_{ji}(t)}^{t}\dot{\mathbf{x}}_{1j}(\sigma)d\sigma \leq& \frac{c_i}{2}|\dot{\mathbf{x}}_{1i}|^2 + \frac{1}{2c_i}\left|\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{x}}_{1j}(\sigma)d\sigma\right|^2 \\ \leq& \frac{c_i}{2}|\dot{\mathbf{x}}_{1i}|^2 + \frac{\overline{T}_{ji}}{2c_i}\int_{t-T_{ji}(t)}^{t}|\dot{\mathbf{x}}_{1j}(\sigma)|^2 d\sigma\end{aligned}$$

on the other hand, invoking Lemma 6.1 from [42] we obtain, under Assumption **A1**

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_{1i} - \mathbf{x}_{1j})^\top(\dot{\mathbf{x}}_{1i} + \dot{\mathbf{x}}_{1j}) = 0.$$

Hence, we obtain

$$\sum_{i=1}^{N}\left(\dot{\mathbf{x}}_{1i}^\top \mathbf{e}_i + \dot{\mathcal{W}}_i\right) \leq \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}\left(c_i|\dot{\mathbf{x}}_{1i}|^2 + \frac{\overline{T}_{ji}^2}{c_i}|\dot{\mathbf{x}}_{1j}|^2\right).$$

This motivates us to define the function

$$\mathcal{E} := \sum_{i=1}^{N}\left(\frac{1}{k_{pi}}\mathcal{U}_i + \mathcal{W}_i\right) \tag{17}$$

which in view of all the previous calculations has a time derivative that satisfies the following bound:

$$\begin{aligned}\dot{\mathcal{E}} \leq& -\sum_{i=1}^{N}\left(\frac{k_{Ii}\phi_i}{k_{pi}}|\dot{\mathbf{x}}_{1i}|^2 + \frac{k_{Ii}\mu_i}{k_{pi}}|\tilde{\boldsymbol{\xi}}_i|^2\right) \\ & + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}\left(c_i|\dot{\mathbf{x}}_{1i}|^2 + \frac{\overline{T}_{ji}^2}{c_i}|\dot{\mathbf{x}}_{1j}|^2\right).\end{aligned}$$

Following the same procedure as in [43], leveraging the fact that the Laplacian matrix is symmetric and thus $a_{ij} = a_{ji}$, we can combine the first term of the first summation with the corresponding terms in the second summation. Hence, we can further write this bound as

$$\dot{\mathcal{E}} \leq -\sum_{i=1}^{N}\left(\chi_i|\dot{\mathbf{x}}_{1i}|^2 + \frac{k_{Ii}\mu_i}{k_{pi}}|\tilde{\boldsymbol{\xi}}_i|^2\right)$$

where

$$\chi_i := \frac{k_{Ii}\phi_i}{k_{pi}} - \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}\left(c_i + \frac{\overline{T}_{ij}^2}{c_j}\right).$$

Therefore, setting k_{Ii} such that (15) holds, ensures that $\chi_i > 0$, for all $i \in \bar{N}$, and hence $\dot{\mathcal{E}} \leq 0$.

Since $\mathcal{E} \geq 0$ and $\dot{\mathcal{E}} \leq 0$, then $\dot{\mathbf{x}}_{1i}, \tilde{\boldsymbol{\xi}}_i \in \mathcal{L}_2$. Moreover, $\mathcal{E}$ is radially unbounded with regards to $\mathbf{x}_{2i}, \tilde{\boldsymbol{\xi}}_i, |\mathbf{x}_{1i} - \mathbf{x}_{1j}|$. This, and the fact that $\dot{\mathcal{E}} \leq 0$, implies that $\mathbf{x}_{2i}, \tilde{\boldsymbol{\xi}}_i, |\mathbf{x}_{1i} - \mathbf{x}_{1j}| \in \mathcal{L}_\infty$, for all $i \in \bar{N}$ and $j \in \mathcal{N}$.

Using the error equation in (12), we can establish that $\dot{\mathbf{x}}_{1i} \in \mathcal{L}_2$ and $|\mathbf{x}_{1i} - \mathbf{x}_{1j}| \in \mathcal{L}_\infty$ ensure that $\mathbf{e}_i \in \mathcal{L}_\infty$. To see this in more detail note that

$$\begin{aligned}\left|\int_{t-T_{ji}(t)}^{t}\dot{\mathbf{x}}_{1j}(\sigma)d\sigma\right|^2 \leq& T_{ji}(t)\int_{t-T_{ji}(t)}^{t}|\dot{\mathbf{x}}_{1j}(\sigma)|^2 d\sigma \\ \leq& \overline{T}_{ji}\int_{0}^{\infty}|\dot{\mathbf{x}}_{1j}(\sigma)|^2 d\sigma\end{aligned}$$

where to obtain the first inequality, we used Schwarz's inequality [41], and to obtain the second inequality we used Assumption **A3**. The last bound is precisely the square of the $\mathcal{L}_2$-norm of $\dot{\mathbf{x}}_{1j}$. Thus, since all the signals in the right-hand side of the closed loop (14) are bounded, then $\dot{\mathbf{x}}_{2i}, \dot{\tilde{\boldsymbol{\xi}}}_i \in \mathcal{L}_\infty$.

From the fact that $\tilde{\boldsymbol{\xi}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{\tilde{\boldsymbol{\xi}}}_i \in \mathcal{L}_\infty$, we can establish, using Barbalat's Lemma [41], that

$$\lim_{t\to\infty}\tilde{\boldsymbol{\xi}}_i(t) = \mathbf{0}.$$

After the differentiation of the first equation of (14), we conclude that also $\ddot{\mathbf{x}}_{1i} \in \mathcal{L}_\infty$. Hence, Barbalat's Lemma allows to conclude that

$\lim_{t\to\infty} \dot{\mathbf{x}}_{1i}(t) = \mathbf{0}$. This also implies, from (14) that $\lim_{t\to\infty} \mathbf{x}_{2i}(t) = \mathbf{0}$.

Therefore, $\lim_{t\to\infty} \int_0^t \dot{\mathbf{x}}_{2i}(\sigma)d\sigma = -\mathbf{x}_{2i}(0)$. Furthermore, under Assumptions **A1–A3**, the time-derivative of the right-hand side of the closed loop (14) is also bounded. Thus, $\ddot{\mathbf{x}}_{2i} \in \mathcal{L}_\infty$. As a consequence, Barbalat's Lemma also ensures that $\lim_{t\to\infty} \dot{\mathbf{x}}_{2i}(t) = \mathbf{0}$. Thus, $\lim_{t\to\infty 0} \mathbf{e}_i = \mathbf{0}$, as required. □

V. Euler–Lagrange Case

In this section, we show that the proposed smooth controller can solve the consensus problem for multiple perturbed EL systems with time delays and without requiring any knowledge on the constant input disturbance, which is still an open problem.

The Lagrangian function for each agent is given by

$$\mathcal{L}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) := \frac{1}{2}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i - \mathcal{P}_i(\mathbf{q}_i)$$

where $\mathbf{q}_i \in \mathbb{R}^n$ and $\dot{\mathbf{q}}_i \in \mathbb{R}^n$ are the generalized positions and velocities, respectively [6]. Matrix $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and it is positive definite and bounded for all $\mathbf{q}_i \in \mathbb{R}^n$. The EL equations of motion are

$$\frac{d}{dt}\nabla_{\dot{\mathbf{q}}_i}\mathcal{L}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) - \nabla_{\mathbf{q}_i}\mathcal{L}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \mathbf{u}_i + \mathbf{d}_i$$

and these can be written in compact form as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \nabla\mathcal{P}_i(\mathbf{q}_i) = \mathbf{u}_i + \mathbf{d}_i \tag{18}$$

where $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i := \dot{\mathbf{M}}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i - \frac{1}{2}\nabla_{\mathbf{q}_i}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$.

Using the momentum definition $\mathbf{p}_i = \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$, together with the Legendre transformation, the Hamiltonian function of these agents can be computed as $\bar{H}_i := \mathbf{p}_i^\top \dot{\mathbf{q}}_i - \mathcal{L}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$, which in turn yields the typical pH system

$$\begin{aligned}\dot{\mathbf{q}}_i &= \nabla_{\mathbf{p}_i}\bar{H}_i\\ \dot{\mathbf{p}}_i &= -\nabla_{\mathbf{q}_i}\bar{H}_i + \mathbf{u}_i + \mathbf{d}_i.\end{aligned}$$

Now, mimicking the results in [44], we perform a change of coordinates given by

$$(\mathbf{x}_{1i}, \mathbf{x}_{2i}) \mapsto (\mathbf{q}_i, \Psi_i(\mathbf{q}_i)\mathbf{p}_i) \tag{19}$$

where matrix $\Psi_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ comes from the factorization of the inertia matrix as

$$\mathbf{M}_i^{-1}(\mathbf{q}_i) = \Psi_i(\mathbf{q}_i)\Psi_i(\mathbf{q}_i) \tag{20}$$

which is well defined. Therefore

$$\begin{aligned}\dot{\mathbf{x}}_{1i} &= \mathbf{M}_i^{-1}(\mathbf{q}_i)\mathbf{p}_i\\ &= \Psi_i(\mathbf{x}_{1i})\nabla_{\mathbf{x}_{2i}}H_i\end{aligned}$$

where to obtain these equations, we have employed the change of coordinates (19) and the Hamiltonian function expressed in these new coordinates, which is given by

$$H_i = \frac{1}{2}|\mathbf{x}_{2i}|^2 + \mathcal{P}_i(\mathbf{x}_{1i}). \tag{21}$$

For the dynamics of $\mathbf{x}_{2i}$, we have that

$$\begin{aligned}\dot{\mathbf{x}}_{2i} &= \dot{\Psi}_i(\mathbf{q}_i)\mathbf{p}_i + \Psi_i(\mathbf{q}_i)\dot{\mathbf{p}}_i\\ &= \dot{\Psi}_i(\mathbf{q}_i)\Psi_i^{-1}(\mathbf{q}_i)\Psi_i^{-1}(\mathbf{q}_i)\dot{\mathbf{x}}_{1i} - \Psi_i(\mathbf{q}_i)\left(\nabla_{\mathbf{q}_i}H_i - \mathbf{u}_i - \mathbf{d}_i\right)\\ &= \Psi_i(\mathbf{x}_{1i})\left(\mathbf{u}_i + \mathbf{d}_i - \nabla_{\mathbf{x}_{1i}}\bar{H}_i\right) + J_i(\mathbf{x}_{1i}, \mathbf{x}_{2i})\nabla_{\mathbf{x}_{2i}}\bar{H}_i\end{aligned}$$

where we have used, again, the momentum definition and then the result in [44, Remark 1], with the (l, k)th elements of the matrix $J_i = [J_{ilk}] \in \mathbb{R}^{n\times n}$ given by

$$J_{ilk}(\mathbf{x}_{1i}, \mathbf{x}_{2i}) = -{\mathbf{x}_{2i}}^\top \Psi_i^{-1}[\Psi_{il}, \Psi_{ik}]$$

$[\Psi_{il}, \Psi_{ik}]$ represents the standard Lie bracket between the column vectors Ψ_{il} and Ψ_{ik}, respectively.

Therefore, using the proposed change of coordinates, we can write the EL systems (18) as in (1) with

$$\Phi_i(\mathbf{x}_{1i}) = \Omega_i(\mathbf{x}_{1i}) := \Psi_i(\mathbf{x}_{1i}), \quad \mathcal{M}_i = I_n$$

with Hamiltonian (21). In this case, the control law (13) yields

$$\begin{aligned}\mathbf{u}_i &= \nabla\mathcal{P}_i(\mathbf{x}_{1i}) - k_{pi}\mathbf{e}_i - k_{di}\mathbf{x}_{2i} - k_{Ii}\boldsymbol{\xi}_i\\ \dot{\boldsymbol{\xi}}_i &= k_{pi}\Psi_i^\top(\mathbf{x}_{1i})\mathbf{e}_i + \left[k_{ri}\Psi_i(\mathbf{x}_{1i}) - J_i(\mathbf{x}_{1i}, \mathbf{x}_{2i})\right]\mathbf{x}_{2i}.\end{aligned} \tag{22}$$

The following result is a corollary of Theorem 1 applied to EL systems, whose proof follows the same steps as in the proof of Theorem 1.

Corollary 1: Consider the systems (18) controlled by (22) with the change of coordinates (19). Suppose that Assumptions **A2** and **A3** hold and that the inertia matrix can be parameterized as (20). Set the gains as $k_{ri} = 5k_{Ii}$ and $k_{di} = 3k_{Ii}$. In addition, ensure that condition (15) holds. Then, for all $\mathbf{q}_i(0), \dot{\mathbf{q}}_i(0) \in \mathbb{R}^n$

$$\lim_{t\to\infty} \mathbf{q}_i(t) = \mathbf{q}_c; \qquad \lim_{t\to\infty} \dot{\mathbf{q}}_i(t) = \mathbf{0}.$$

◁

VI. Simulations

In this section, we validate our proposed robust control approach through simulations with five 2-degrees-of-freedom robot manipulators, whose inertial matrix and geometrical parameters are the same for all robots.

The inertia matrix of these systems has the form

$$\mathbf{M} = \begin{bmatrix}\theta_1 + \theta_2\cos(q_2) & \theta_3 + \theta_2\cos(q_2)\\ \theta_3 + \theta_2\cos(q_2) & \theta_3\end{bmatrix} = \begin{bmatrix}m_{11} & m_{12}\\ m_{12} & m_{22}\end{bmatrix}$$

with potential energy

$$\mathcal{P}(\mathbf{q}) = \theta_4(1 - \cos(q_1)) + \theta_5(1 - \cos(q_1 + q_2))$$

and parameters $\theta_1 := \frac{1}{2}(m_1 l_{c1}^2 + m_2 l_1^2 + I_1) = 1.82$, $\theta_2 := m_2 l_1 l_{c2} = 0.29$, $\theta_3 := \frac{1}{2}(m_2 l_{c2}^2 + I_2) = 0.51$, $\theta_4 := (m_1 l_{c1} + m_2 l_1)g = 48.18$, and $\theta_5 := m_2 l_{c2} g = 13.21$.

To simplify the notation, subindex i has been omitted from all signals and parameters in these definitions.

The inverse of the inertia matrix is given by

$$\mathbf{M}^{-1} = \frac{1}{\Delta}\begin{bmatrix}m_{22} & -m_{12}\\ -m_{12} & m_{11}\end{bmatrix} \tag{23}$$

where $\Delta = m_{11}m_{22} - m_{12}^2$. Using the well-known analytic square root matrix decomposition, $\Psi(\mathbf{q})$ takes the form

$$\Psi = \frac{1}{\sqrt{\varrho}}\begin{bmatrix}m_{22} + \sqrt{\Delta} & -m_{12}\\ -m_{12} & m_{11} + \sqrt{\Delta}\end{bmatrix} \tag{24}$$

with $\varrho = \Delta(m_{22} + m_{11} + 2\sqrt{\Delta})$.

The interaction between the robots is represented by the connected undirected graph, as shown in Fig. 1. The control gains are set to $k_{pi} = 12$ and $k_{Ii} = 30$, which verify (15) for all $i \in \{1, \ldots, 5\}$. The time-varying bounded delays $T_{ji}(t)$ were generated randomly with a normal distribution of mean μ =0.2, variance $\sigma^2 = 0.001$, and a sample time 0.05 s. The latter implies that the communication delays between each pair of connected robots are different. Fig. 2 illustrates

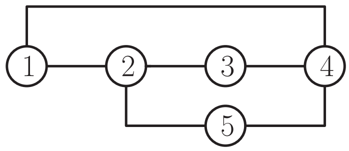

Fig. 1. Communication topology: undirected connected graph.

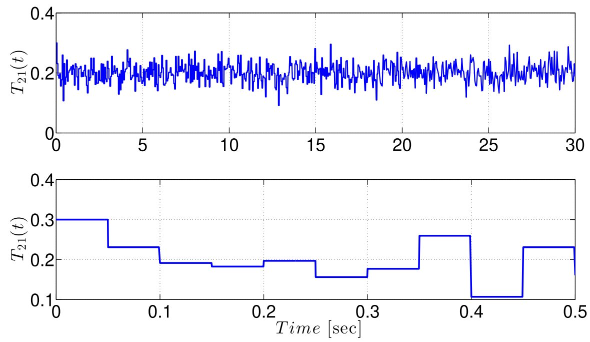

Fig. 2. Variable delay perceived by the robot 1 for the information received from neighbor 2.

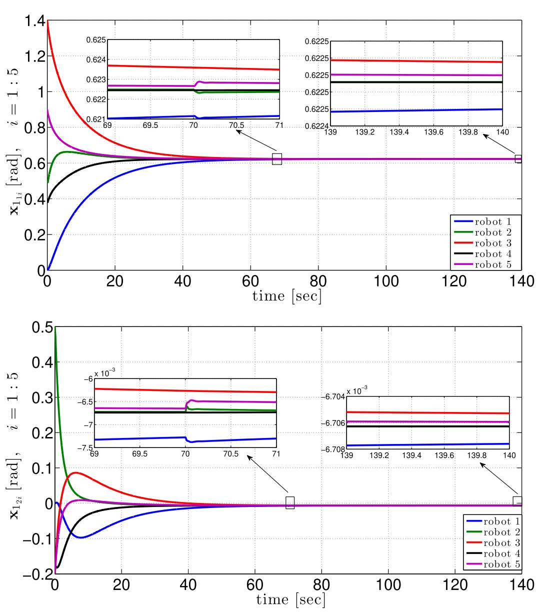

Fig. 3. Positions $\mathbf{x}_{1_{1i}}$ and $\mathbf{x}_{1_{2i}}$ of all agents.

the delay for perceived by agent 1 for the incoming data from agent 2. In this simulation, the delay is not differentiable, since this is the type of delays usually found in practice, e.g., in communication via WiFi. Note, however, that despite these nonsmooth delays not satisfying Assumption **A3**, the controller is still able to make the vehicles reach the desired formation, which leads us to conjecture that the technical assumption of boundedness of the derivative of the delay may not be needed.

To assess the robustness of the controller proposed, we consider step changes in the disturbances affecting the robot manipulators 1, 2, and 5. These values commute after 70 s from d^+_{ki} to $d^\star_{ki}$, with $k \in \{1, 2\}$, and they are appreciated in Table I, where we also show the initial position for each robot of the network. In addition, we set to zero all initial velocities $\mathbf{x}_{2i}(0)$ and all initial values $\xi_i(0)$.

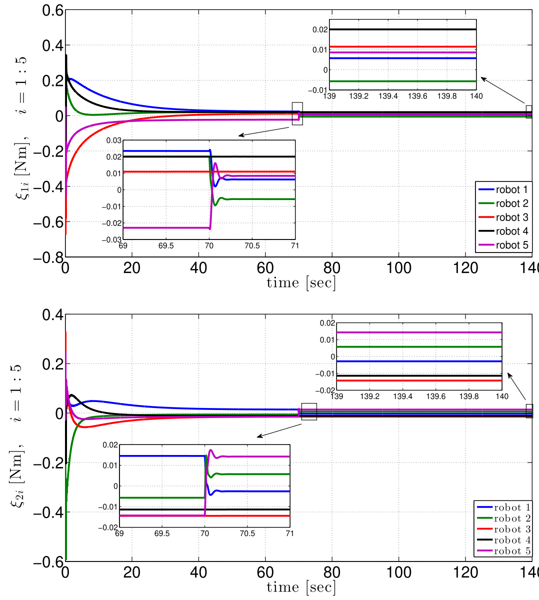

Fig. 4. Integral action ξ_{1i} and ξ_{2i}.

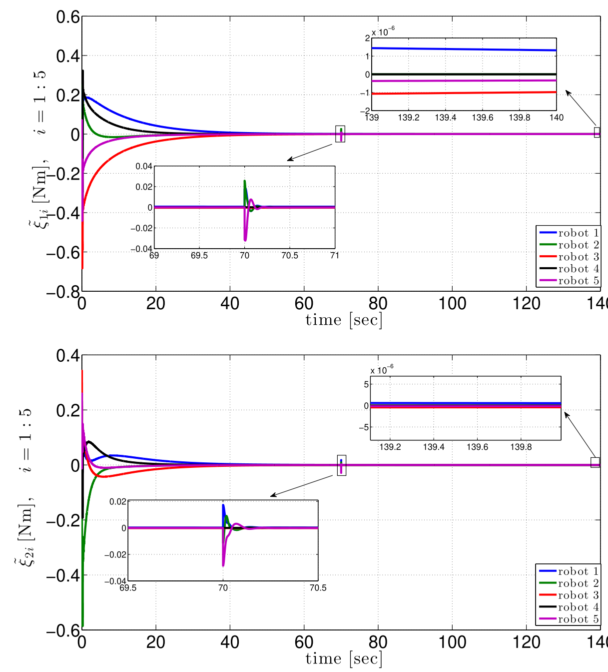

Fig. 5. Integral action errors $\tilde{\xi}_{1i}$ and $\tilde{\xi}_{2i}$.

TABLE I
INITIAL CONDITIONS AND DISTURBANCES

	q_{1i}	q_{2i}	d^+_{1i}	d^+_{2i}	$d^\star_{1i}$	$d^\star_{2i}$
1	0	0	-0.8	0.5	0.2	-0.1
2	0.5	0.5	0.7	-0.2	-0.2	0.2
3	1.4	-0.2	0.4	-0.5	0.4	-0.5
4	0.4	-0.2	0.7	-0.4	0.7	-0.4
5	0.9	-0.2	-0.8	-0.5	0.3	0.5

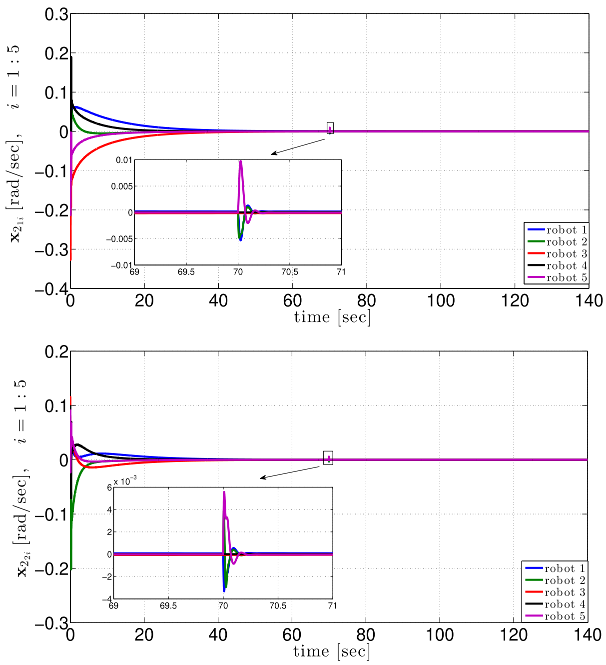

Fig. 6. Transformed velocities $\mathbf{x}_{2_{1i}}$ and $\mathbf{x}_{2_{2i}}$.

Fig. 3 shows the time history of the robots' position coordinates. Moreover, since the robots are affected by the step changes disturbances at time 70 s, we notice that the consensus points are not altered. This fact allows us to see the robustness of the proposed controller under disturbances. Furthermore, this good performance is due to the faster disturbance rejection achieved by the new integral action $\boldsymbol{\xi}_i$ as we can appreciate in Figs. 4 and 5, where also the convergence to zero of $\tilde{\boldsymbol{\xi}}_i$ has been established—even in the presence of the step changes of the disturbances. Finally, Fig. 6 shows the convergence to zero of the velocity coordinates.

VII. Concluding Remarks and Future Work

In this work, we establish the solution to the robust consensus problem for a class of perturbed pH systems. The solution comes from a novel controller that has the structure of a P+d injection with an integral term. Provided that the integral gain is sufficiently large, we show that the agents find a consensus at a common agreement point. The result is robust to constant external disturbances and to varying time-delays in the communications. A byproduct of our methodology is the solution to the robust consensus of EL systems using a simple to implement PID controller.

The main avenues for future work are: 1) to include nonholonomic restrictions in the pH dynamics; 2) to consider time-varying disturbances; 3) to deal with parameter uncertainty; and 4) to work with directed graph topologies. All these issues are, to the best of the authors' knowledge, open problems for agents whose nonlinear dynamics can be represented as pH systems.

References

[1] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.
[2] A. Jadbabaie, J. Lin, and S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, Jun. 2003.
[3] F. Chen and W. Ren, *Distributed Average Tracking in Multi-Agent Systems*. Berlin, Germany: Springer, 2020.
[4] J. G. Romero, A. Donaire, and R. Ortega, "Robust energy shaping control of mechanical systems," *Syst. Control Lett.*, vol. 62, pp. 770–780, 2013.
[5] S. I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*. London, U.K.: Springer, 2001.
[6] T. Hatanaka, N. Chopra, M. Fujita, and M. Spong, *Passivity-Based Control and Estimation in Networked Robotics*. Berlin, Germany: Springer, 2015.
[7] W. Ren, "Distributed leaderless consensus algorithms for networked Euler-Lagrange systems," *Int. J. Control*, vol. 82, no. 11, pp. 2137–2149, 2009.
[8] S. -J. Chung and J. -J. Slotine, "Cooperative robot control and concurrent synchronization of Lagrangian systems," *IEEE Trans. Robot.*, vol. 25, no. 3, pp. 686–700, Jun. 2009.
[9] E. Nuño, R. Ortega, L. Basañez, and D. Hill, "Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays," *IEEE Trans. Autom. Control*, vol. 56, no. 4, pp. 935–941, Apr. 2011.
[10] E. Nuño, I. Sarras, and L. Basañez, "Consensus in networks of nonidentical Euler–Lagrange systems using P + D controllers," *IEEE Trans. Robot.*, vol. 26, no. 6, pp. 1503–1508, Dec. 2013.
[11] C. De Persis and B. Jayawardhana, "On the internal model principle in the coordination of nonlinear systems," *IEEE Trans. Control Netw. Syst.*, vol. 1, no. 3, pp. 272–282, Sep. 2014.
[12] E. Nuño, I. Sarras, H. Yin, and B. Jayawardhana, "Consensus of Euler–Lagrange agents with internal model disturbance rejection and interconnection delays," *IEEE Trans. Autom. Control*, vol. 69, no. 6, pp. 4066–4071, Jun. 2024.
[13] M. Lu and L. Liu, "Robust synchronization control of switched networked Euler–Lagrange systems," *IEEE Trans. Cybern.*, vol. 52, no. 7, pp. 6834–6842, Jul. 2022.
[14] H. Jian, S. Zheng, P. Shi, Y. Xie, and H. Li, "Consensus for multiple random mechanical systems with applications on robot manipulator," *IEEE Trans. Ind. Electron.*, vol. 71, no. 1, pp. 846–856, Jan. 2024.
[15] M. Ye, B. D. O. Anderson, and C. Yu, "Leader tracking of Euler–Lagrange agents on directed switching networks using a model-independent algorithm," *IEEE Trans. Control Netw. Syst.*, vol. 6, no. 2, pp. 561–571, Jun. 2019.
[16] Z. Feng, G. Hu, W. Ren, W. E. Dixon, and J. Mei, "Distributed coordination of multiple unknown Euler–Lagrange systems," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 1, pp. 55–66, Mar. 2018.
[17] M. Long and H. Su, "Model-independent robust consensus of multiple Euler–Lagrange systems," *IEEE Trans. Control Netw. Syst.*, vol. 10, no. 1, pp. 368–380, Mar. 2023.
[18] A. Rosales, Y. Shtessel, L. Fridman, and C. B. Panathula, "Chattering analysis of HOSM controlled systems: Frequency domain approach," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4109–4115, Aug. 2017.
[19] J. G. Romero, E. Nuño, and C. Aldana, "Global consensus-based formation control of perturbed nonholonomic mobile robots with time varying delays," *Int. J. Franklin Inst.*, vol. 361, no. 2, pp. 557–571, 2024.
[20] J. G. Romero, D. Navarro-Alarcón, E. Nuño, and H. Que, "A globally convergent adaptive velocity observer for nonholonomic mobile robots affected by unknown disturbances," *IEEE Control Syst. Lett.*, vol. 7, pp. 85–90, 2023.
[21] T. Hatanaka, N. Chopra, T. Ishizaki, and N. Li, "Passivity-based distributed optimization with communication delays using PI consensus algorithm," *IEEE Trans. Autom. Control*, vol. 63, no. 12, pp. 4421–4428, Dec. 2018.
[22] A. Murat, "Passivity as a design tool for group coordination," *IEEE Trans. Autom. Control*, vol. 8, no. 52, pp. 1380–1390, Aug. 2007.
[23] N. Li, P. Borja, M. A. Scherpen, A. van der Schaft, and R. Mahony, "Passivity-based trajectory tracking and formation control of nonholonomic wheeled robots without velocity measurements," *IEEE Trans. Automat. Control*, vol. 12, no. 68, pp. 7951–7957, Dec. 2023.
[24] F. Yue and D. Zelazo, "A passivity analysis for nonlinear consensus on balanced digraphs," *Eur. J. Control*, vol. 85, Sep. 2025, Art. no. 101303.
[25] E. Nuño and R. Ortega, "Achieving consensus of Euler–Lagrange agents with interconnecting delays and without velocity measurements via passivity-based control," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 1, pp. 222–232, Jan. 2018.
[26] A. van der Schaft and B. Maschke, "Port-Hamiltonian systems on graphs," *SIAM J. Control Optim.*, vol. 51, no. 2, pp. 906–937, 2013.
[27] S. Feng, Y. Kawano, M. Cucuzzella, and J. Scherpen, "Output consensus control for linear port-Hamiltonian systems," in *Proc. Int. Symp. Math. Theory Netw. Syst.*, 2022, pp. 230–235.
[28] V. Duindam, A. Macchelli, S. Stramigioli, and H. Bruyninckx, *Modeling and Control of Complex Physical Systems*. Berlin, Germany: Springer, 2009.
[29] A. van der Schaft and D. Jeltsema, *Port-Hamiltonian Systems Theory: An Introductory Overview*. Hanover, USA: Now Publishers Inc, 2014.

[30] J. E. Marsden and T. S. Ratiu, *Introduction to Mechanics and Symmetry*. New York, NY, USA: Springer, 1999.

[31] A. Donaire and T. Perez, “Dynamic positioning of marine craft using a port-Hamiltonian framework,” *Automatica*, vol. 48, no. 5, pp. 851–856, 2012.

[32] A. Donaire, J. G. Romero, and T. Perez, “Trajectory tracking passivity-based control for marine vehicles subject to disturbances,” *J. Franklin Inst.*, vol. 354, no. 5, pp. 2167–2182, 2017.

[33] J. Alcaraz-Tapia, C. Castañeda, and H. Vargas-Rodriguez, “Port-Hamiltonian mathematical model of a fluid ring attitude system,” *Energies*, vol. 14, no. 21, 2021, Art. no. 6906.

[34] N. Javanmardi, A. Yaghmaei, and M. Yazdanpanah, “Spacecraft formation flying in the port-Hamiltonian framework,” *Nonlinear Dyn.*, vol. 99, no. 4, pp. 2765–2783, 2020.

[35] G. Flores, A. Montes de Oca, and A. Flores, “Robust nonlinear control for the fully actuated hexa-rotor: Theory and experiments,” *IEEE Control Syst. Lett.*, vol. 7, pp. 277–282, 2023.

[36] J. G. Romero, D. Navarro-Alarcon, and E. Panteley, “Robust globally exponentially stable control for mechanical systems in free/constrained-motion tasks,” in *Proc. IEEE Conf. Decis. Control*, 2013, pp. 3067–3072.

[37] W. M. Haddad and V. Chellaboina, *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*. New Jersey, USA: Princeton Univ. Press, 2008.

[38] A. Abdessameud, I. Polushin, and A. Tayebi, “Synchronization of lagrangian systems with irregular communication delays,” *IEEE Trans. Autom. Control*, vol. 59, no. 1, pp. 187–193, Jan. 2014.

[39] A. Abdessameud, I. G. Polushin, and A. Tayebi, “Distributed coordination of dynamical multi-agent systems under directed graphs and constrained information exchange,” *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 1668–1683, Apr. 2017.

[40] E. Nuño, I. Sarras, A. Loría, M. Maghenem, E. Cruz-Zavala, and E. Panteley, “Strict Lyapunov–Krasovskii functionals for undirected networks of Euler–Lagrange systems with time-varying delays,” *Syst. Control Lett.*, vol. 135, 2020, Art. no. 104579.

[41] H. Khalil, *Nonlinear Systems*. New Jersey, USA: Prentice-Hall, 2002.

[42] W. Ren and Y. Cao, *Distributed Coordination of Multi-Agent Networks: Emergent Problems, Models, and Issues*. London, U.K.: Springer, 2011.

[43] E. Nuño, “Consensus of Euler–Lagrange systems using only position measurements,” *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 1, pp. 489–498, Mar. 2018.

[44] A. Venkatraman, R. Ortega, I. Sarras, and A. van der Schaft, “Speed observation and position feedback stabilization of partially linearizable mechanical systems,” *IEEE Trans. Autom. Control*, vol. 55, no. 5, pp. 1059–1074, May 2010.